

Lepton flavour violating B meson decays via scalar leptoquark

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Abstract

We study the effect of scalar leptoquarks in the lepton flavour violating B meson decays induced by the flavour changing transitions $b \rightarrow ql_i^+ l_j^-$ with $q = s, d$. In the standard model these transitions are extremely rare as they are either two-loop suppressed or proceed via box diagrams with tiny neutrino masses in the loop. However, in the leptoquark model they can occur at tree level and are expected to have significantly large branching ratios. The leptoquark parameter space is constrained using the experimental limits on the branching ratios of $B_q \rightarrow l^+ l^-$ processes. Using such constrained parameter space, we predict the branching ratios of LFV semileptonic B meson decays, such as $B^+ \rightarrow K^+(\pi^+)l_i^+ l_j^-$, $B^+ \rightarrow (K^{*+}, \rho^+)l_i^+ l_j^-$ and $B_s \rightarrow \phi l_i^+ l_j^-$, which are found to be within the experimental reach of LHCb and the upcoming Belle II experiments. We also investigate the rare leptonic $K_{L,S} \rightarrow \mu^+ \mu^- (e^+ e^-)$ and $K_L \rightarrow \mu^\mp e^\pm$ decays in the leptoquark model.

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I. INTRODUCTION

The study of rare B decay modes induced by the flavour changing neutral current (FCNC) transitions are immensely helpful to test the standard model (SM) and provide hints for new physics beyond it. The SM contributions to the rare B meson decays involving FCNC transitions $b \rightarrow s, d$ are absent at the tree level due to Glashow-Iliopoulos-Maiani (GIM) mechanism and occur via one-loop level only. Although, so far we have not observed any clear indication of new physics in the B sector, but there are several observables measured by the BaBar, Belle, CDF and LHCb Collaborations in the semileptonic B decays involving the transition $b \rightarrow sl^+l^-$, have significant deviations from the corresponding SM predictions. Specifically, the observation of 3σ anomaly by the LHCb experiments in the $B \rightarrow K^{*0}\mu^+\mu^-$ decay rate [1, 2] and in the angular observable P'_5 [3] have attracted a lot of attention in recent times. In addition, lepton flavour non-universality has been observed in the ratio of $B \rightarrow K\mu\mu$ to $B \rightarrow Kee$ branching fractions [4] and the decay rate of $B_s \rightarrow \phi\mu^+\mu^-$ process [5] also deviates from the SM predictions by about 3σ .

On the other hand, the observation of neutrino oscillations has provided unambiguous evidence for lepton number violation in the neutral lepton sector, even though the individual lepton number is conserved in the SM of the electroweak interaction. The observation of the neutrino masses and mixing and the violation of family lepton number could in principle allow FCNC transitions in the charged lepton sector as well, such as $l_i \rightarrow l_j\gamma$, $l_i \rightarrow l_j l_k \bar{l}_k$, $B \rightarrow l_i^\pm l_j^\mp$ and $B \rightarrow K^{(*)} l_i^\pm l_j^\mp$ etc. It is interesting to see if these branching ratios could be enhanced in some new physics model which could simultaneously explain the observed anomalies. As LHCb has already reported violation of lepton universality in the $B \rightarrow Kl^+l^-$ process having deviation from the SM prediction by 2.6σ , which in turn hints towards the possibility of observing lepton flavour violating (LFV) decays also. As pointed out in Ref. [6], a possible explanation for the observed LHCb data on R_K , i.e., the lepton non-universality is due to 25% deficit in the muon channel, which implies LFV is larger for muons than for electrons. The LFV decays in the charged lepton sector has been studied in various new physics model in the literature [7, 8]. Even though there is no direct experimental evidence for such processes, but there exist severe constraints on some of these LFV modes [9]. The experimental observation of lepton flavour violating decays would provide unambiguous signal of new physics beyond the SM.

The most elegant ways to look for new physics in FCNC processes are the prudent investigation of the anomalies associated with $b \rightarrow sl^+l^-$ decays observed at LHCb [1–5]. These anomalies have been studied in the SM and in various extensions of it [10–13]. In this paper we are interested to investigate the lepton flavour violating B meson decays such as $B^+ \rightarrow K^+(\pi^+)l_i^+l_j^-$, $B^+ \rightarrow (K^{*+}, \rho^+)l_i^+l_j^-$, $B_s \rightarrow \phi l_i^+l_j^-$, $K_{L,S} \rightarrow \mu^-\mu^+(e^-e^+)$ and $K_L \rightarrow \mu^\pm e^\mp$ in the scalar leptoquark model. These decay processes are extremely rare in the SM as they are either two-loop suppressed or proceed through box diagrams with the presence of tiny neutrino masses in the loop. However, in the leptoquark model they can occur at the tree level and hence, can give observable signature in the LHCb experiment. Leptoquarks are color triplet gauge particles having both baryon and lepton quantum numbers and can be either scalars or vectors. Even though leptoquarks do not address some important open questions like the dark matter content of the Universe or the origin of the electroweak scale, these particles allow quark-lepton transitions at tree level and thus, point towards the theory of quark-lepton universality. Leptoquarks can come from the extended Standard models [14] which treat quarks and leptons in equal footing such as Grand unified model [14, 15], Pati-Salam model, quark and lepton composite models [16], extended technicolor model [17] etc. Leptoquarks having baryon and lepton number violating couplings are very massive to avoid proton decay or large Majorana neutrino masses. However, the baryon and lepton number conserving leptoquarks could be light enough to be accessible in accelerator searches and also they do not induce proton decay. In the literature there are many attempts [8, 18–24] to explain the observed anomalies in the leptoquark model.

The outline of the paper is follows. The effective Hamiltonian describing the $b \rightarrow ql^+l^-$, $q = s, d$ processes is briefly discussed in section II. In section III we present the new physics contribution due to the scalar leptoquark exchange and the constraint on leptoquark parameter space by using the experimental limit on the branching ratios of the rare decays $B_q \rightarrow l^+l^-$. The branching ratios for LFV decays $B^+ \rightarrow P^+l_i^+l_j^-$, $P = K, \pi$ and $B_{(s)}^+ \rightarrow V^+(\phi)l_i^+l_j^-$, $V = K^*, \rho$ decays in the leptoquark model are presented in sections IV and V respectively. In section VI we compute the branching ratios of rare $K_{L,S} \rightarrow \mu^+\mu^-(e^+e^-)$ decays and the LFV decays $K_L \rightarrow \mu^\mp e^\pm$ are investigated in section VII. Section VIII contains the summary and conclusion.

II. THE EFFECTIVE HAMILTONIAN FOR $b \rightarrow (s, d)l^+l^-$ PROCESS

The effective Hamiltonian mediating the rare semileptonic decay $b \rightarrow ql^+l^-$, $q = s, d$ in the standard model is [25, 26]

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} \left[\lambda_t^{(q)} \mathcal{H}_{eff}^{(t)} + \lambda_u^{(q)} \mathcal{H}_{eff}^{(u)} \right] + h.c., \quad (1)$$

where

$$\begin{aligned} \mathcal{H}_{eff}^{(u)} &= C_1(\mathcal{O}_1^c - \mathcal{O}_1^u) + C_u(\mathcal{O}_2^c - \mathcal{O}_2^u), \\ \mathcal{H}_{eff}^{(t)} &= C_1\mathcal{O}_1^c + C_2\mathcal{O}_2^c + \sum_{i=3}^{10} C_i\mathcal{O}_i, \end{aligned} \quad (2)$$

and $\lambda_{q'}^{(q)} = V_{q'b}V_{q'q}^*$ ($q' = t, u$) are the product of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Here \mathcal{O}_i 's are the most general dimension-six flavour changing operators and $C_{i=1,\dots,10}$ are their respective Wilson coefficients evaluated at renormalization scale $\mu = m_b$ [26]. The sum over i corresponds to the tree level current-current operators ($\mathcal{O}_{1,2}$), the QCD penguin operators \mathcal{O}_{3-6} , the photon and gluon dipole operators $\mathcal{O}_{7,8}^{(\prime)}$ and the semileptonic operators $\mathcal{O}_{9,10}^{(\prime)}$ which can be expressed as

$$\begin{aligned} \mathcal{O}_7^{(\prime)} &= \frac{e}{16\pi^2} \left[\bar{s}\sigma_{\mu\nu}(m_s P_{L(R)} + m_b P_{R(L)})b \right] F^{\mu\nu}, \\ \mathcal{O}_9^{(\prime)} &= \frac{\alpha}{4\pi} (\bar{s}\gamma^\mu P_{L(R)}b) (\bar{l}\gamma_\mu l), \quad \mathcal{O}_{10}^{(\prime)} = \frac{\alpha}{4\pi} (\bar{s}\gamma^\mu P_{L(R)}b) (\bar{l}\gamma_\mu \gamma_5 l). \end{aligned} \quad (3)$$

It should be noted that the primed operators which have opposite chirality to the unprimed ones are negligible in the SM and can only be generated using new physics beyond the SM. The Fermi constant is denoted by G_F , α is the fine-structure constant and $P_{L,R} = (1 \mp \gamma_5)/2$ are the chiral operators. For $b \rightarrow s$ transitions, the contribution of $\mathcal{H}_{eff}^{(u)}$ is doubly Cabibbo-suppressed with respect to that of $\mathcal{H}_{eff}^{(t)}$ due to the CKM factor $V_{ub}V_{us}^*$ and can be neglected. However, for $b \rightarrow d$ transition, $\lambda_t^{(d)}$ and $\lambda_u^{(d)}$ are comparable in magnitude with a sizable phase difference, hence, in addition with $\mathcal{H}_{eff}^{(t)}$, the decay amplitude from $\mathcal{H}_{eff}^{(u)}$ is also relevant. In the next section, we will discuss the new physics contribution to the SM effective Hamiltonian due to the exchange of scalar leptoquark and constrain the product of various leptoquark couplings from some rare B decays.

III. NEW PHYSICS CONTRIBUTIONS DUE TO SCALAR LEPTOQUARK EXCHANGE

There will be additional contributions to the SM effective Hamiltonian (1) in the scalar leptoquark model due to the exchange of LQ's between the external fermion particles. As discussed in [8, 19], out of all possible leptoquark multiplets which are invariant under the SM gauge group $SU(3) \times SU(2) \times U(1)$ the two scalar leptoquark multiplets $X = (3, 2, 7/6)$ and $X = (3, 2, 1/6)$ do not allow proton decay. These scalar leptoquarks can have sizable Yukawa couplings and could potentially contribute to the quark level transition $b \rightarrow ql^+l^-$. Due to the chirality and diagonality nature and the conservation of both baryon and lepton number, these leptoquarks may provide an interesting testing ground to look for their effects in rare B meson decays.

In the scalar LQ model, the Lagrangian describing the interaction of the scalar leptoquark doublet $X = (3, 2, 7/6)$ with the charged leptons is given by [19]

$$\mathcal{L} = -\lambda_u^{ij} \bar{u}_R^i X^T \epsilon L_L^j - \lambda_e^{ij} \bar{e}_R^i X^\dagger Q_L^j + h.c. , \quad (4)$$

where i, j are the generation indices, Q_L and L_L are the left handed quark and lepton doublets, u_R and e_R are the right handed up-type quark and charged lepton singlets and $\epsilon = i\sigma_2$ is a 2×2 matrix. These multiplets can be represented more explicitly as

$$X = \begin{pmatrix} V_\alpha \\ Y_\alpha \end{pmatrix}, \quad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \text{and} \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (5)$$

After expanding the $SU(2)$ indices the interaction Lagrangian becomes

$$\mathcal{L} = -\lambda_u^{ij} \bar{u}_{\alpha R}^i (V_\alpha e_L^j - Y_\alpha \nu_L^j) - \lambda_e^{ij} \bar{e}_R^i (V_\alpha^\dagger u_{\alpha L}^j + Y_\alpha^\dagger d_{\alpha L}^j) + h.c. . \quad (6)$$

After performing the Fierz transformation in (6) and then comparing it with the SM effective Hamiltonian (1), one can obtain the new Wilson coefficients to the $b \rightarrow ql_i^+ l_i^-$ processes as

$$C_9^{NP} = C_{10}^{NP} = -\frac{\pi}{2\sqrt{2}G_F\alpha V_{tb}V_{tq}^*} \frac{\lambda_e^{i3}\lambda_e^{ik*}}{M_Y^2}, \quad (7)$$

where k is the generation index of the quark flavor q . Analogously, the Lagrangian for the coupling of the scalar leptoquark $X = (3, 2, 1/6)$ to the fermion bilinear is

$$\mathcal{L} = -\lambda_d^{ij} \bar{d}_{\alpha R}^i (V_\alpha e_L^j - Y_\alpha \nu_L^j) + h.c. , \quad (8)$$

which provides additional contributions to the primed semileptonic electroweak penguin operators $\mathcal{O}'_{9,10}$ and their corresponding new primed Wilson coefficients $C'^{NP}_{9,10}$ are given as

$$C'^{NP}_9 = -C'^{NP}_{10} = \frac{\pi}{2\sqrt{2} G_F \alpha V_{tb} V_{tq}^*} \frac{\lambda_d^{ki} \lambda_d^{3i*}}{M_V^2}. \quad (9)$$

Here the superscript index k represents the generation of the down type quark, i.e., $k = 1$ or 2 , depending upon the coupling of leptoquark to d or s . The subscript $d(e)$ in the leptoquark couplings of Eqns. (7) and (9) stands for all families of down-type quarks (charged leptons). These new Wilson coefficients $C'^{(\nu)NP}_{9,10}$ or in other words, the leptoquark parameters can be constrained by comparing the theoretical [27] and experimental [28–30] branching ratios of $B_q \rightarrow \mu^+ \mu^-$ processes. The detailed formalism of the constraints on leptoquark coupling has been discussed in [8, 21], therefore here we will simply quote the results.

$$1.5 \times 10^{-9} \text{ GeV}^{-2} \leq \frac{|\lambda_e^{23} \lambda_e^{21*}|}{M_S^2} = \frac{|\lambda_d^{32*} \lambda_d^{12}|}{M_S^2} \leq 3.9 \times 10^{-9} \text{ GeV}^{-2}, \quad (10)$$

$$0 \leq \frac{|\lambda_e^{23} \lambda_e^{22*}|}{M_S^2} = \frac{|\lambda_d^{32*} \lambda_d^{22}|}{M_S^2} \leq 5 \times 10^{-9} \text{ GeV}^{-2} \quad \text{for} \quad \pi/2 \leq \phi^{NP} \leq 3\pi/2, \quad (11)$$

where M_S is the mass of the scalar leptoquark. Also for simplicity, we have not kept the subscripts on the leptoquark coupling parameters. Analogously from the $B_{s,d} \rightarrow \tau^+ \tau^-$ ($e^+ e^-$) leptonic decays, the constraints on various combination of leptoquark couplings can be obtained by comparing the theoretically predicted branching ratios [27] with the corresponding experimental ones, for which only the upper limits are known [9]. The upper bound on the product of the leptoquark couplings from various two body leptonic $B_{s,d} \rightarrow l^+ l^-$, $l = e, \mu, \tau$ decays are presented in Table-I [8]. In our previous work [8, 22], we studied the effect of scalar leptoquarks on various observables associated with $B \rightarrow K^{(*)} \mu^+ \mu^- (\nu \bar{\nu})$ and $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ processes by using these constraint leptoquark couplings. We found significant deviation in the asymmetry parameters from their SM predictions and thus explains the anomalies observed at LHCb and other B -factories quite well.

IV. $B^+ \rightarrow P^+ l_i^+ l_j^-$

Here, we will discuss the lepton flavour violating semileptonic B mesons decays to pseudoscalar mesons K and π , which are mediated by the $b \rightarrow q l_i^+ l_j^-$ quark level transition.

TABLE I: Constraints obtained from the leptoquark couplings from various leptonic $B_{s,d} \rightarrow l^+ l^-$ decays.

Decay Process	Couplings involved	Upper bound of the couplings (GeV^{-2})
$B_s \rightarrow \mu^\pm \mu^\mp$	$\frac{ \lambda^{23} \lambda^{22*} }{M_S^2}$	$\leq 5 \times 10^{-9}$
$B_s \rightarrow e^\pm e^\mp$	$\frac{ \lambda^{13} \lambda^{12*} }{M_S^2}$	$< 2.54 \times 10^{-5}$
$B_s \rightarrow \tau^\pm \tau^\mp$	$\frac{ \lambda^{33} \lambda^{32*} }{M_S^2}$	$< 1.2 \times 10^{-8}$
$B_d \rightarrow \mu^\pm \mu^\mp$	$\frac{ \lambda^{23} \lambda^{21*} }{M_S^2}$	$(1.5 - 3.9) \times 10^{-9}$
$B_d \rightarrow e^\pm e^\mp$	$\frac{ \lambda^{13} \lambda^{11*} }{M_S^2}$	$< 1.73 \times 10^{-5}$
$B_d \rightarrow \tau^\pm \tau^\mp$	$\frac{ \lambda^{33} \lambda^{31*} }{M_S^2}$	$< 1.28 \times 10^{-6}$

As discussed earlier, these processes occur at tree level due to the exchange of scalar leptoquarks. Fig. 1 depicts the tree level Feynman diagram for the lepton flavour violating process $b \rightarrow s l_i^+ l_j^-$, where leptoquark can couple to a quark and a lepton simultaneously. Analogously, one can obtain the diagram for $b \rightarrow d l_i^+ l_j^-$ process by replacing s with d and incorporating the appropriate LQ couplings. Here i, j denote the lepton family numbers. We will present the results for the scalar LQ $X(3, 2, 7/6)$ and analogously one can obtain the results for $X(3, 2, 1/6)$. Thus, the effective Hamiltonian for $b \rightarrow q l_i^\pm l_j^\mp$ process in the scalar LQ model is given by [8]

$$\mathcal{H}_{LQ} = [G_{LQ} (\bar{q} \gamma^\mu P_L b) (\bar{l}_i \gamma_\mu (1 + \gamma_5) l_j) + H_{LQ} (\bar{q} \gamma^\mu P_L b) (\bar{l}_j \gamma_\mu (1 + \gamma_5) l_i)] , \quad (12)$$

where the constant coefficient G_{LQ} and H_{LQ} are

$$G_{LQ} = \frac{\lambda^{i3} \lambda^{jk*}}{8M_Y^2}, \quad H_{LQ} = \frac{\lambda^{j3} \lambda^{ik*}}{8M_Y^2} . \quad (13)$$

The matrix element of the quark-current between the initial and final mesons can be parameterized in terms of the form factors f_+^P and f_0^P as [31]

$$\langle P(p') | \bar{s} \gamma_\mu b | \bar{B}(p_B) \rangle = (2p_B - q)_\mu f_+^P(q^2) + \frac{M_B^2 - M_P^2}{q^2} q_\mu [f_0^P(q^2) - f_+^P(q^2)] , \quad (14)$$

where $q = p_B - p'$ and $f_{+,0}^P$ correspond to kaon and pion form factors, which are taken from [32] and [33] respectively. The transition amplitudes for $B^+ \rightarrow P^+ l_i^- l_j^+$, $P = K, \pi$ processes

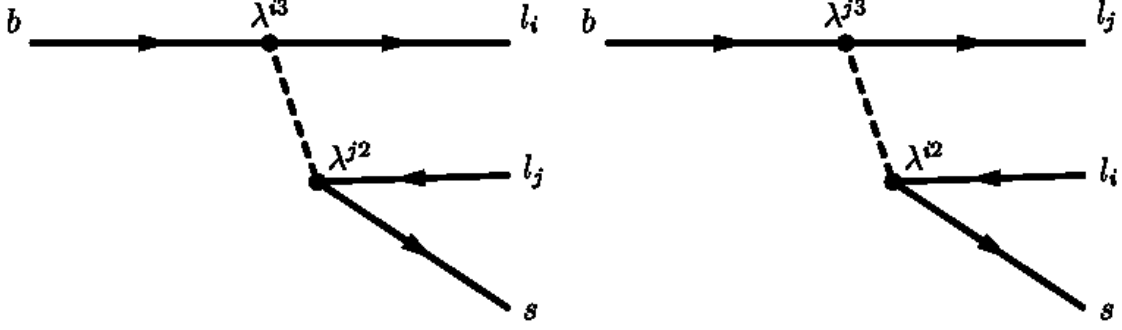


FIG. 1: Feynman diagram for lepton flavour violating $b \rightarrow sl_i^- l_j^+$ process (left panel) and $b \rightarrow sl_i^+ l_j^-$ (right panel) mediated by the scalar leptoquark where $l = e, \mu, \tau$.

are given as

$$\mathcal{M} = \left[F_S(\bar{l}_i l_j) + F_P(\bar{l}_i \gamma_5 l_j) + F_V P^\mu (\bar{l}_i \gamma_\mu l_j) + F_A P^\mu (\bar{l}_i \gamma_\mu \gamma_5 l_j) \right], \quad (15)$$

where

$$\begin{aligned} F_V &= G_{LQ}, & F_A &= G_{LQ}, \\ F_S &= \frac{1}{2} G_{LQ} (m_j - m_i) \left[\frac{M_B^2 - M_P^2}{q^2} \left(\frac{f_0^P(q^2)}{f_+^P(q^2)} - 1 \right) - 1 \right], \\ F_P &= \frac{1}{2} G_{LQ} (m_i + m_j) \left[\frac{M_B^2 - M_P^2}{q^2} \left(\frac{f_0^P(q^2)}{f_+^P(q^2)} - 1 \right) - 1 \right]. \end{aligned} \quad (16)$$

Analogously, the transition amplitude for $B^+ \rightarrow P^+ l_i^+ l_j^-$ process can be obtained from (15) by replacing G_{LQ} by H_{LQ} and $l_i \leftrightarrow l_j$.

Thus, one can obtain the differential decay distribution for the process $B^+ \rightarrow P^+ l_i^+ l_j^-$, with respect to q^2 and $\cos \theta$ as

$$\frac{d\Gamma}{dq^2 d\cos \theta} = a(q^2) + b(q^2) \cos \theta + c(q^2) \cos^2 \theta, \quad (17)$$

where

$$\begin{aligned} a(q^2) &= \Gamma_0 \frac{\sqrt{\lambda_1 \lambda_2}}{q^2} (f_+^P)^2 \left[(|F_V|^2 + |F_A|^2) \frac{\lambda_1}{4} + |F_S|^2 (q^2 - (m_i + m_j)^2) \right. \\ &\quad + |F_P|^2 (q^2 - (m_i - m_j)^2) + |F_A|^2 M_B^2 (m_i + m_j)^2 + |F_V|^2 M_B^2 (m_i - m_j)^2 \\ &\quad \left. + (M_B^2 - M_P^2 + q^2) \left((m_i + m_j) \text{Re}(F_P F_A^*) + (m_j - m_i) \text{Re}(F_S F_V^*) \right) \right], \end{aligned} \quad (18)$$

$$b(q^2) = \Gamma_0 \frac{\sqrt{\lambda_1 \lambda_2}}{q^2} (f_+^P)^2 \left[(m_i + m_j) \text{Re}(F_S F_V^*) - (m_j - m_i) \text{Re}(F_P F_A^*) \right], \quad (19)$$

$$c(q^2) = -\Gamma_0 (f_+^P)^2 \frac{(\lambda_1 \lambda_2)^{3/2}}{4q^6} (|F_A|^2 + |F_V|^2), \quad (20)$$

and

$$\Gamma_0 = \frac{1}{2^8 \pi^3 M_B^3}, \quad \lambda_1 = \lambda(M_B^2, M_P^2, q^2), \quad \lambda_2 = \lambda(q^2, m_i^2, m_j^2),$$

with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac)$. (21)

It should be noted that, in the SM there are no intermediate states which can decay into two leptons belonging to different generations. Therefore, LFV decays have no long distance QCD contributions and no dominant charmonium resonance background like $B \rightarrow K^{(*)} l^+ l^-$ processes. Therefore, the background suppression for these channels would be relatively low.

After obtaining the expression for the branching ratio of $B^+ \rightarrow P^+ l_i^+ l_j^-$ processes, we will proceed for numerical estimations. The particle masses and the life time of B meson are taken from [9]. The scalar leptoquarks are diagonal and have full strength coupling when couples to a lepton and a quark of the same generation. The coupling of LQ with quark and leptons of different generations are assumed to follow Cabibbo-like suppression behavior. It should be noted that the expansion parameter of the CKM matrix in the Wolfenstein parameterization can be related to the down type quark masses as $\lambda \sim (m_d/m_s)^{1/2}$, while in the lepton sector one can have the same order for λ with the relation $\lambda \sim (m_{l_i}/m_{l_j})^{1/4}$. Therefore, in order to compute the required couplings, we used the coupling given in Table-I as basis values and assumed that the leptoquark couplings between different generation of quarks and leptons follow the simple scaling law, i.e. $\lambda^{ij} \simeq (m_i/m_j)^{1/4} \lambda^{ii}$ with $j > i$.

With these input parameters we show in Fig. 2 the variation of branching ratio for lepton flavour violating decays $B^+ \rightarrow K^+ \mu^+ e^-$ (left panel), $B^+ \rightarrow K^+ \tau^+ e^-$ (right panel) and $B^+ \rightarrow K^+ \tau^+ \mu^-$ (bottom panel) with respect to q^2 in the full physical region. In Fig. 3, we have shown the variation of branching ratios of $B^+ \rightarrow \pi^+ \mu^+ e^-$ (left panel), $B^+ \rightarrow \pi^+ \tau^+ e^-$ (right panel) and $B^+ \rightarrow \pi^+ \tau^+ \mu^-$ (bottom panel) processes with respect to q^2 . The blue bands represent the allowed range of the branching ratio of semileptonic LFV decays $B^+ \rightarrow \pi^+ \mu^+ e^- (\tau^+ \mu^-)$ induced by the scalar leptoquarks, as in these cases we have also the lower bound on the leptoquark couplings as seen from Table-I. The predicted branching ratio of $B^+ \rightarrow K^+ (\pi^+) l_i^+ l_j^-$ LFV decays in respective physical range and their

corresponding experimental upper limits are presented in Table-II. The predicted branching ratios are found to be lower than the present experimental upper limits and they are within the reach of LHCb and Belle II experiments.

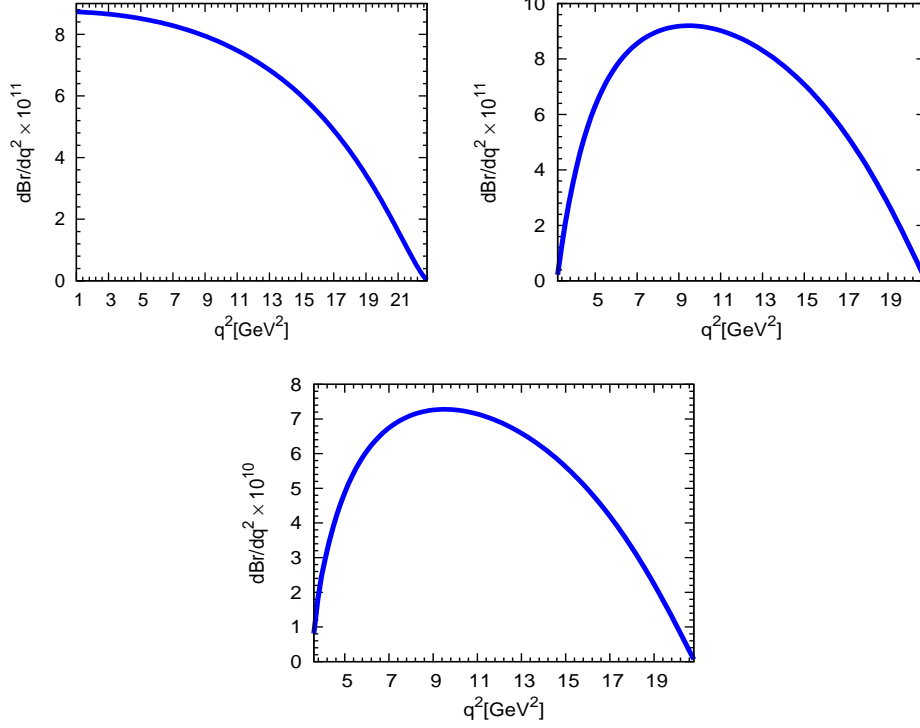


FIG. 2: The variation of branching ratio of $B^+ \rightarrow K^+ \mu^+ e^-$ (left panel), $B^+ \rightarrow K^+ \tau^+ e^-$ (right panel), and $B^+ \rightarrow K^+ \tau^+ \mu^-$ (bottom panel) with respect to q^2 in the scalar leptoquark model.

V. $B^+ \rightarrow V^+ l_i^+ l_j^-$ AND $B_s \rightarrow \phi l_i^+ l_j^-$

In this section we describe the theoretical framework to calculate the branching ratio for the LFV decays $B_{(s)}^+ \rightarrow V^+(\phi) l_i^- l_j^+$, where the vector meson V corresponds to K^*/ρ . Here we will discuss in detail for a particular vector boson, i.e., $V = K^*$ case. However, the same formalism can be applied to other vector mesons with appropriate change in the CKM elements and the mass of the particles involved. The amplitude of $B(p) \rightarrow K^*(k)[\rightarrow K(k_1)\pi(k_2)] l_i^+(p_i) l_j^-(p_j)$ decay mediated via the scalar leptoquark can be obtained from the effective Hamiltonian (12) and is given by

$$\mathcal{M} = G_{LQ} \langle K\pi | \bar{s} \gamma_\mu (1 - \gamma_5) b | B \rangle (\bar{l}_i \gamma^\mu (1 + \gamma_5) l_j) . \quad (22)$$

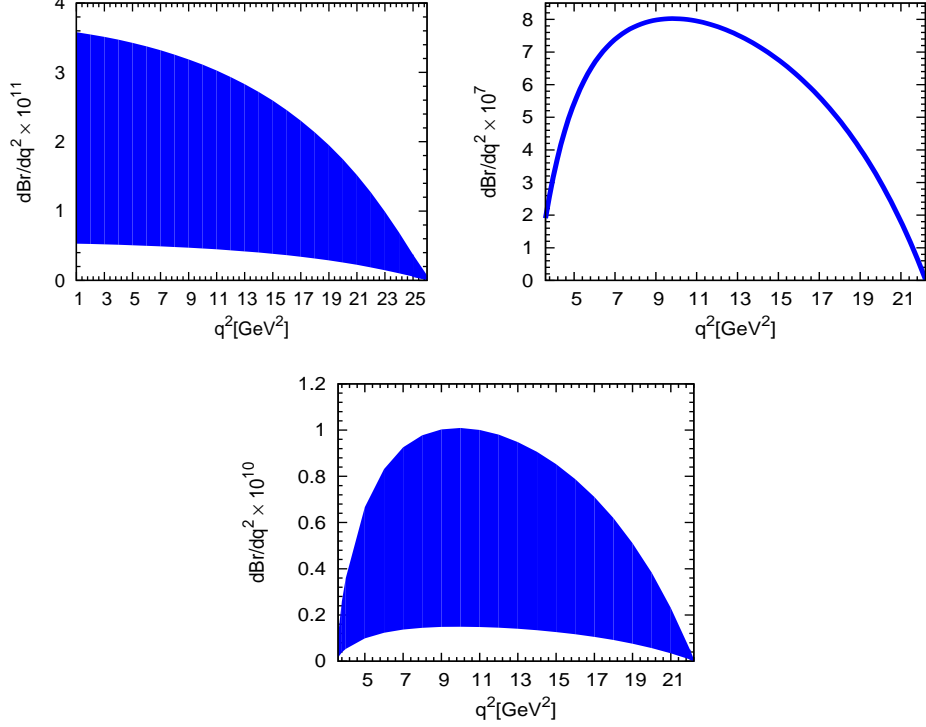


FIG. 3: The variation of branching ratio of $B^+ \rightarrow \pi^+ \mu^+ e^-$ (left panel), $B^+ \rightarrow \pi^+ \tau^+ e^-$ (right panel), and $B^+ \rightarrow \pi^+ \tau^+ \mu^-$ (bottom panel) with respect to q^2 in the scalar leptoquark model.

This amplitude can be expressed in terms of $B \rightarrow K^*$ form factors by assuming that K^* decays resonantly. The $B \rightarrow K^*$ hadronic matrix elements of the local quark bilinear operators can be parametrized as [34]

$$\begin{aligned} \langle K^*(k) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle = & \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha q^\beta \frac{2V(q^2)}{M_B + M_{K^*}} - i \epsilon_\mu^* (M_B + M_{K^*}) A_1(q^2) \\ & + i (\epsilon^* \cdot q) (2p - q)_\mu \frac{A_2(q^2)}{M_B + M_{K^*}} + i \frac{2M_{K^*}}{q^2} (\epsilon^* \cdot q) [A_3(q^2) - A_0(q^2)] q_\mu, \end{aligned} \quad (23)$$

where q^2 is the momentum transfer between the B and K^* meson, i.e., $q_\mu = p_\mu - k_\mu$ and ϵ_μ is the polarization vector of the K^* meson. In the narrow width approximation the squared K^* propagator can be expressed as

$$\frac{1}{(k^2 - M_{K^*}^2)^2 + (M_{K^*} \Gamma_{K^*})^2} \xrightarrow{\Gamma_{K^*} \ll M_{K^*}} \frac{\pi}{M_{K^*} \Gamma_{K^*}} \delta(k^2 - M_{K^*}^2). \quad (24)$$

One can avoid the $K^* K \pi$ coupling $g_{K^* K \pi}$ in the $B \rightarrow K \pi$ amplitude as it cancels with the vertex factor and the width of K^* meson

$$\Gamma_{K^*} = \frac{g_{K^* K \pi}^2}{48\pi} M_{K^*} \beta^3, \quad (25)$$

TABLE II: The predicted branching ratios for $B^+ \rightarrow K^+(\pi^+)l_i^+l_j^-$ lepton flavour violating decays, where $l = e, \mu, \tau$ in the scalar LQ $X(3, 2, 7/6)$ model.

Decay process	Predicted BR	Experimental limit [9]
$B^+ \rightarrow K^+\mu^+e^-$	$<1.36 \times 10^{-9}$	$<1.3 \times 10^{-7}$
$B^+ \rightarrow K^+\tau^+\mu^-$	$<8.8 \times 10^{-9}$	$<2.8 \times 10^{-5}$
$B^+ \rightarrow K^+\tau^+e^-$	$<1.12 \times 10^{-9}$	$<1.5 \times 10^{-5}$
$B^+ \rightarrow \pi^+\mu^+e^-$	$(0.91 - 6.16) \times 10^{-10}$	$<6.4 \times 10^{-3}$
$B^+ \rightarrow \pi^+\tau^+\mu^-$	$(0.18 - 1.2) \times 10^{-9}$	$<4.5 \times 10^{-5}$
$B^+ \rightarrow \pi^+\tau^+e^-$	$<9.65 \times 10^{-6}$	$<2.0 \times 10^{-5}$

where

$$\beta = \frac{1}{M_{K^*}^2} [M_{K^*}^4 + M_K^4 + M_\pi^4 - 2(M_{K^*}^2 M_K^2 + M_K^2 M_\pi^2 + M_{K^*}^2 M_\pi^2)]^{1/2}. \quad (26)$$

If one writes symbolically the $B \rightarrow K^*$ matrix elements (23) as

$$\langle K^*(k) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle = \epsilon^{*\nu} A_{\nu\mu}, \quad (27)$$

where $A_{\nu\mu}$ contains the $B \rightarrow K^*$ form factors, then $B \rightarrow K\pi$ matrix element can be expressed as

$$\langle K\pi | \bar{s} \gamma_\mu (1 - \gamma_5) b | B \rangle = -D_{K^*}(k^2) \left[K^\nu - \frac{M_K^2 - M_\pi^2}{k^2} k^\nu \right] A_{\nu\mu}, \quad (28)$$

with

$$|D_{K^*}(k^2)|^2 = g_{K^*K\pi}^2 \frac{\pi}{M_{K^*} \Gamma_{K^*}} \delta(k^2 - M_{K^*}^2) = \frac{48\pi^2}{\beta^3 M_{K^*}^2} \delta(k^2 - M_{K^*}^2). \quad (29)$$

In our analysis, we have used the following symmetric and antisymmetric combination of momentum as

$$k = k_1 + k_2, \quad K = k_1 - k_2, \quad q = p_i + p_j, \quad Q = p_i - p_j. \quad (30)$$

The full angular distribution of $B \rightarrow K^* l_i^+ l_j^-$ decay can be completely described by the four independent kinematic variables, the dilepton invariant mass squared q^2 , the angle ϕ

between the normals to the $K\pi$ and the dilepton ($l_i^+ l_j^-$) planes in the rest frame of the B meson and the angles θ_K and θ_l . The physical region of the phase space are

$$(m_i + m_j)^2 \leq q^2 \leq (M_B - M_{K^*})^2, \quad -1 \leq \cos \theta_l \leq 1, \quad -1 \leq \cos \theta_K \leq 1, \quad 0 \leq \phi \leq 2\pi. \quad (31)$$

If we integrate out all the three angles θ_K , θ_l and ϕ in their respective kinematically accessible physical range, we will get the differential decay rate with respect to the dilepton mass squared q^2 , which is given by

$$\begin{aligned} \frac{d\Gamma}{dq^2} = & \Gamma_V \times \left[A(q^2)^2 \left\{ \frac{2}{3} \lambda_{K^*} \left(1 - \left(\frac{m_i^2}{q^2} \right)^2 \right) + 8M_{K^*}^2 (q^2 - m_i^2) \right. \right. \\ & \left. \left. - \frac{2}{9} \left(1 - \frac{m_i^2}{q^2} \right)^2 \left((M_B^2 - M_{K^*}^2 - q^2)^2 + 8q^2 M_{K^*}^2 \right) \right\} \right. \\ & + B(q^2)^2 \left\{ \frac{\lambda_{K^*}}{6} (M_B^2 - M_{K^*}^2 - q^2)^2 \left(1 - \left(\frac{m_i^2}{q^2} \right)^2 \right) - \frac{\lambda_{K^*}^2}{18} \left(1 - \frac{m_i^2}{q^2} \right)^2 \right. \\ & \left. \left. - \frac{2}{3} \lambda_{K^*} M_{K^*}^2 (q^2 - m_i^2) \right\} + C(q^2)^2 \left\{ \frac{2}{3} \lambda_{K^*} m_i^2 (q^2 - m_i^2) \right\} \right. \\ & \left. - D(q^2)^2 \left\{ \frac{4}{9} \lambda_{K^*} M_{K^*}^2 (q^2 - m_i^2) \left(4 - \frac{m_i^2}{q^2} \right) \right\} \right. \\ & \left. - Re \left(A(q^2) B(q^2)^* \right) \left\{ \frac{2}{3} \lambda_{K^*} (M_B^2 - M_{K^*}^2 - q^2) \left(1 - \left(\frac{m_i^2}{q^2} \right)^2 - \frac{1}{3} \left(1 - \frac{m_i^2}{q^2} \right)^2 \right) \right\} \right. \\ & \left. - Re \left(A(q^2) C(q^2)^* \right) \left\{ \frac{4}{3} \lambda_{K^*} m_i^2 \left(1 - \frac{m_i^2}{q^2} \right) \right\} \right. \\ & \left. + Re \left(B(q^2) C(q^2)^* \right) \left\{ \frac{2}{3} \lambda_{K^*} m_i^2 (M_B^2 - M_{K^*}^2 - q^2) \left(1 - \frac{m_i^2}{q^2} \right) \right\} \right], \quad (32) \end{aligned}$$

where

$$\Gamma_V = \frac{3\sqrt{\lambda_{K^*}}}{2^{11} M_{K^*}^2 (\pi M_B \beta)^3} |G_{LQ}|^2, \quad \lambda_{K^*} = \lambda(M_B^2, M_{K^*}^2, q^2), \quad (33)$$

and

$$\begin{aligned} A(q^2) &= (M_B + M_{K^*}) A_1(q^2), & B(q^2) &= \frac{2A_2(q^2)}{(M_B + M_{K^*})}, \\ C(q^2) &= \frac{A_2(q^2)}{(M_B + M_{K^*})} + \frac{2M_{K^*}}{q^2} (A_3(q^2) - A_0(q^2)), & D(q^2) &= \frac{2V(q^2)}{(M_B + M_{K^*})}. \end{aligned} \quad (34)$$

For simplicity, we have neglected the mass of kaon, pion, muon and electron. Here m_i represents the mass of the tau lepton for the LFV decays having tau as a final particle. The

form factors A_i and V are scale independent and are taken from [35], which are valid in the full physical regime. Using the above expressions, the variation of differential branching ratios of $B^+ \rightarrow K^{*+}\mu^+e^-$ (left panel), $B^+ \rightarrow K^{*+}\tau^+e^-$ (right panel) and $B^+ \rightarrow K^{*+}\tau^+\mu^-$ (bottom panel) processes with respect to q^2 are shown in Fig. 4. The predicted branching ratios of the $B^+ \rightarrow K^{*+}l_i^+l_j^-$ processes in the full kinematical regime are presented in Table-III. It is found that the obtained value of $B^+ \rightarrow K^{*+}\mu^+e^-$ branching ratio is within the experimental limit. But so far there exist no experimental upper limits for the $\text{Br}(B^+ \rightarrow K^{*+}\tau^+\mu(e)^-)$ decay processes.

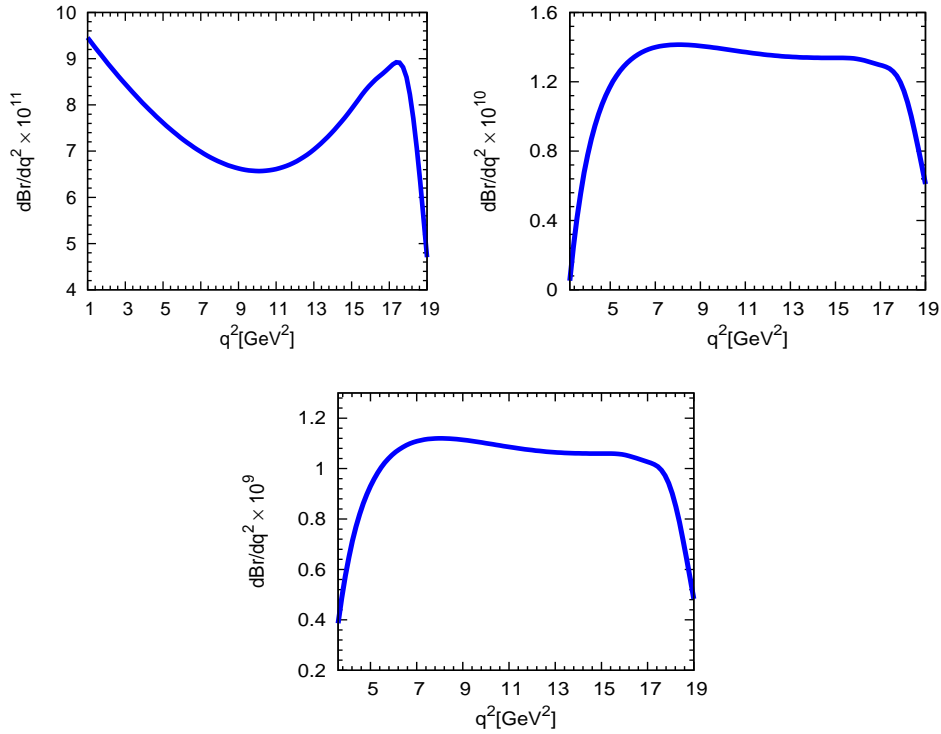


FIG. 4: The variation of branching ratio of $B^+ \rightarrow K^{*+}\mu^+e^-$ (left panel), $B^+ \rightarrow K^{*+}\tau^+e^-$ (right panel), and $B^+ \rightarrow K^{*+}\tau^+\mu^-$ (bottom panel) with respect to q^2 in the scalar leptoquark model.

Similarly we have estimated branching ratios of the $B_s \rightarrow \phi l_i^+ l_j^-$ and $B^+ \rightarrow \rho^+ l_i^+ l_j^-$ processes. For numerical calculation, we have taken the values of form factors of $B_q \rightarrow \phi(\rho)$ from [35] and the particle masses from [9]. Fig. 5 shows the differential branching ratios of $B_s \rightarrow \phi\mu^+e^-$ (left panel), $B_s \rightarrow \phi\tau^+e^-$ (right panel) and $B_s \rightarrow \phi\tau^+\mu^-$ (bottom panel) processes with respect to dilepton invariant mass squared and the branching ratios of $B^+ \rightarrow \rho^+\mu^+e^-$ (left panel), $B^+ \rightarrow \rho^+\tau^+e^-$ (right panel) and $B^+ \rightarrow \rho^+\tau^+\mu^-$ (bottom panel) with q^2

are shown in Fig. 6. The integrated branching ratios of the $B_s \rightarrow \phi l_i^+ l_j^-$ process in the range $(m_i + m_j)^2$ to $(M_{B_s} - M_\phi)^2$ are presented in Table-III. Similarly the predicted branching ratio of $B^+ \rightarrow \rho^+ l_i^+ l_j^-$ up to full range $(M_B - M_\rho)^2 \simeq 20.2 \text{ GeV}^2$ has been presented in Table-III. The experimental upper limit of $B^+ \rightarrow \rho^+ e^\pm \mu^\mp$ process is $< 3.2 \times 10^{-6}$ [36].

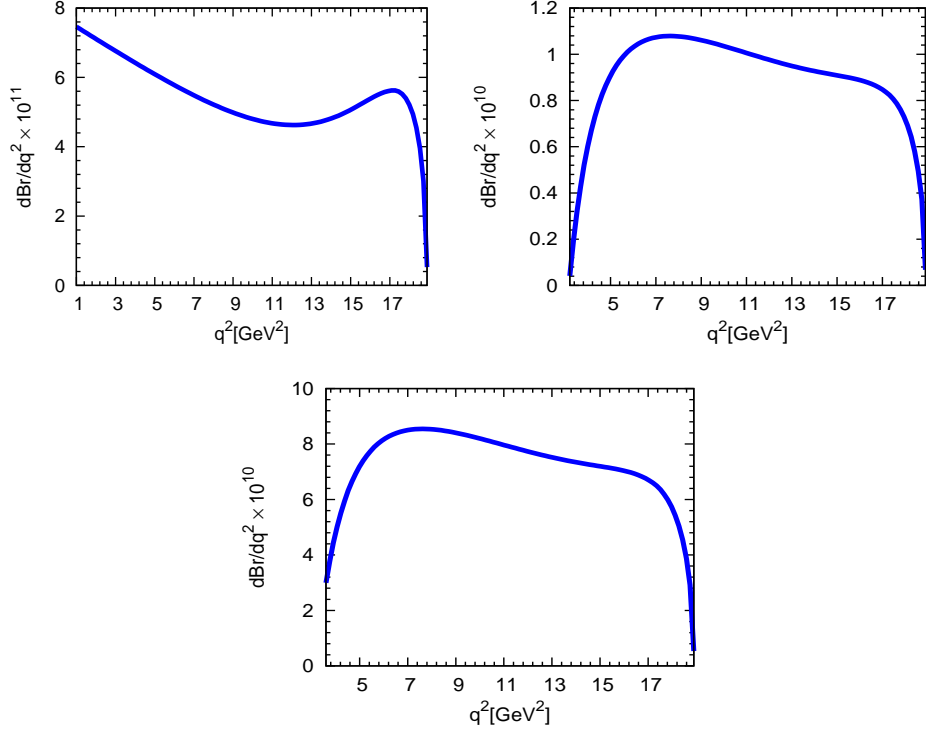


FIG. 5: The variation of branching ratio of $B_s \rightarrow \phi \mu^+ e^-$ (left panel), $B_s \rightarrow \phi \tau^+ e^-$ (right panel), and $B_s \rightarrow \phi \tau^+ \mu^-$ (bottom panel) with respect to q^2 in the scalar leptoquark model.

Recently LHCb has observed 2.6σ discrepancy from the SM prediction in the measurement of the ratio of branching fractions of $B \rightarrow K l^+ l^-$ decays into dimuons over dielectrons in the dilepton invariant mass bin $(1 \leq q^2 \leq 6) \text{ GeV}^2$ [4]. Analogously, we would like to see whether it would be possible to observe the lepton non-universality effects in semileptonic LFV decays. We define the ratio of branching ratios of LFV $B^+ \rightarrow P^+ l_i^+ l_j^-$ decays to $B^+ \rightarrow P^+ \mu^+ e^-$ process as

$$R_{P\mu e}^{l_i l_j} = \frac{\text{BR}(B^+ \rightarrow P^+ l_i^+ l_j^-)}{\text{BR}(B^+ \rightarrow P^+ \mu^+ e^-)}, \quad (35)$$

where $l_{i,j}$ stand for all charged leptons. In Fig. 7, we show the q^2 variation of $R_{K\mu e}^{\tau e}$ (top-left panel), $R_{K\mu e}^{\tau \mu}$ (top-right panel), $R_{K^* \mu e}^{\tau e}$ (bottom-left panel) and $R_{K^* \mu e}^{\tau \mu}$ (bottom-right panel) in the $X = (3, 2, 7/6)$ LQ model. The integrated values of these ratios in

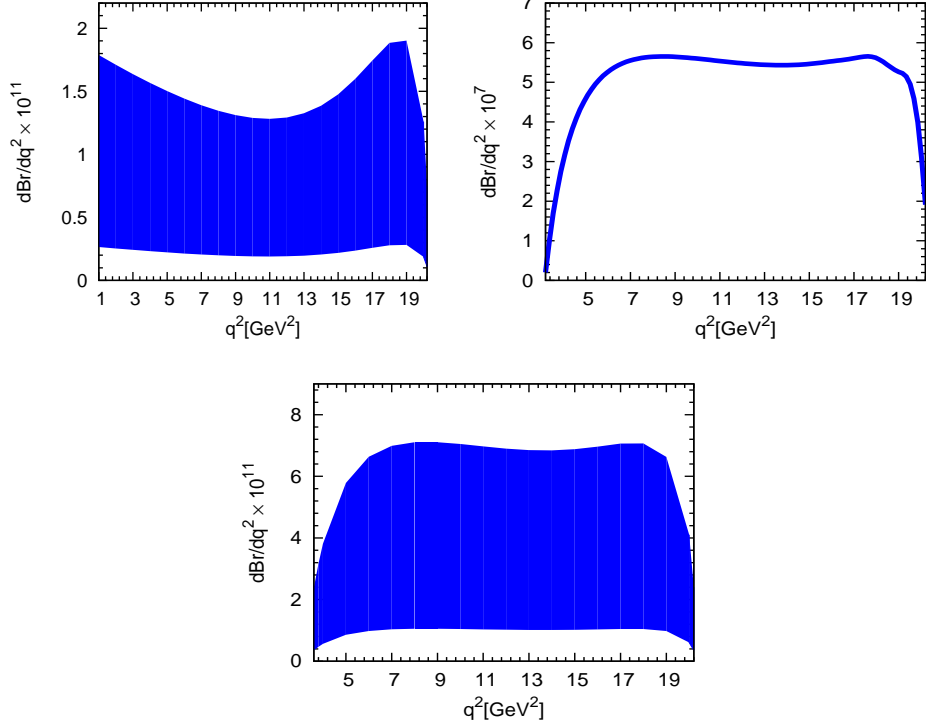


FIG. 6: The variation of branching ratio of $B^+ \rightarrow \rho^+ \mu^+ e^-$ (left panel), $B^+ \rightarrow \rho^+ \tau^+ e^-$ (right panel), and $B^+ \rightarrow \rho^+ \tau^+ \mu^-$ (bottom panel) with respect to q^2 in the scalar leptoquark model.

the $X = (3, 2, 7/6)$ leptoquark model for both scalar and vector meson are presented in Table IV. Here we have considered only the upper limit value for the LQ couplings. Since the couplings of leptoquarks differ when they couple to different generations of quarks and leptons, no definitive conclusion can be inferred from these results. However, if these ratios will be measured in future, they will provide interesting insight about the nature of new physics.

In addition, the ratio of branching fractions of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ over $B^+ \rightarrow K^+ \mu^+ \mu^-$ process has been measured by LHCb collaborations [37] as

$$\frac{\text{BR}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)}{\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)} = 0.053 \pm 0.014(\text{stat}) \pm 0.001 (\text{syst}). \quad (36)$$

Analogously, one can also define the ratio of branching fractions of $B^+ \rightarrow \pi^+ l_i^+ l_j^-$ and $B^+ \rightarrow K^+ l_i^+ l_j^-$ LFV processes as

$$R_{\pi K}^{l_i l_j} = \frac{\text{BR}(B^+ \rightarrow \pi^+ l_i^+ l_j^-)}{\text{BR}(B^+ \rightarrow K^+ l_i^+ l_j^-)}. \quad (37)$$

TABLE III: The predicted branching ratios for $B_{(s)}^+ \rightarrow V^+(\phi)l_i^+l_j^-$ lepton flavour violating decays, where $V = K^*\rho$ and $l = e, \mu, \tau$.

Decay process	Predicted BR	Experimental limit [9]
$B^+ \rightarrow K^{*+}\mu^+e^-$	$<1.4 \times 10^{-9}$	$<9.9 \times 10^{-7}$
$B^+ \rightarrow K^{*+}\tau^+\mu^-$	$<1.56 \times 10^{-8}$...
$B^+ \rightarrow K^{*+}\tau^+e^-$	$<2 \times 10^{-9}$...
$B_s \rightarrow \phi\mu^+e^-$	$<8.2 \times 10^{-10}$...
$B_s \rightarrow \phi\tau^+\mu^-$	$<1.1 \times 10^{-8}$...
$B_s \rightarrow \phi\tau^+e^-$	$<1.42 \times 10^{-9}$...
$B^+ \rightarrow \rho^+\mu^+e^-$	$(0.43 - 2.9) \times 10^{-10}$...
$B^+ \rightarrow \rho^+\tau^+\mu^-$	$(0.162 - 1.1) \times 10^{-9}$...
$B^+ \rightarrow \rho^+\tau^+e^-$	$<8.73 \times 10^{-6}$...

The variation of $R_{\pi K}^{\mu e}$ (left panel), $R_{\rho K^*}^{\mu e}$ (right panel) and $R_{\rho\phi}^{\mu e}$ (bottom panel) with respect to q^2 are shown in Fig. 8. We present the predicted values of above defined ratios in Table IV, (where we have considered only the upper limit value for the LQ couplings), along with the values for the corresponding vector meson case. The study of the above ratios in the leptoquark model provides additional new observables, which could be searched at LHCb and other B -factories.

TABLE IV: The predicted integrated values of $R_{P(V)\mu e}^{l_i l_j}$ and $R_{P P'(V V')}^{l_i l_j}$ observables in the $X = (3, 2, 7/6)$ LQ model, where $P, P' = K, \pi, V, V' = K^*, \phi, \rho$ and $l = e, \mu, \tau$.

Observables	Predicted values	Observables	Predicted values
$R_{K\mu e}^{\tau\mu}$	6.47	$R_{\pi K}^{\mu e}$	0.453
$R_{K\mu e}^{\tau e}$	0.82	$R_{\pi K}^{\tau\mu}$	0.14
$R_{K^*\mu e}^{\tau e}$	1.43	$R_{\rho K^*}^{\tau\mu}$	0.07
$R_{\pi\mu e}^{\tau\mu}$	1.95	$R_{\rho K^*}^{\mu e}$	0.21
$R_{\rho\mu e}^{\tau\mu}$	3.79	$R_{\rho\phi}^{\mu e}$	0.354
$R_{\phi\mu e}^{\tau e}$	1.73	$R_{\rho\phi}^{\tau\mu}$	0.1

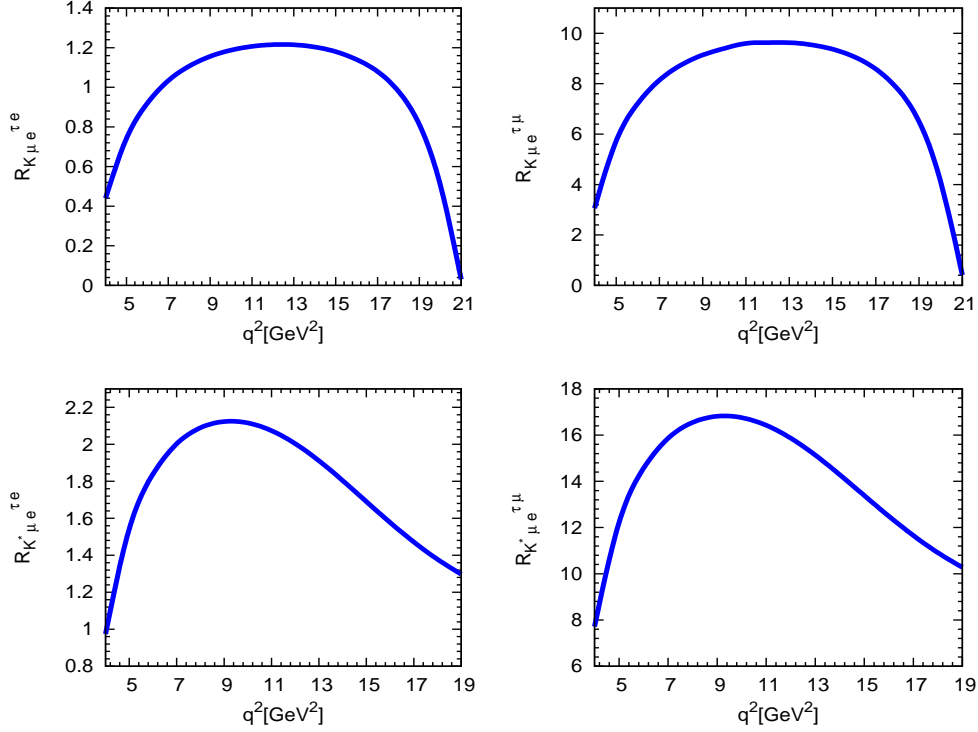


FIG. 7: The variation of observables $R_{K\mu e}^{\tau e}$ (top-left panel), $R_{K\mu e}^{\tau\mu}$ (top-right panel), $R_{K^*\mu e}^{\tau e}$ (bottom-left panel), $R_{K^*\mu e}^{\tau\mu}$ (bottom-right panel), with respect to q^2 in the scalar leptoquark model $X(3, 2, 7/6)$.

VI. $K_{L,S} \rightarrow \mu^+ \mu^- (e^+ e^-)$

In this section, we study the rare leptonic decays of K meson and would like to see how the scalar leptoquarks affect these processes. The rare $K_L \rightarrow \mu^+ \mu^-$ decay is CP conserving and provides valuable information on the short distance physics of $|\Delta S = 1|$ FCNC transitions. This decay mode acquires dominant contributions from the long distance two photon intermediates state $K_L \rightarrow \gamma^* \gamma^* \rightarrow \mu^+ \mu^-$. Therefore, although its branching ratio is measured precisely with value $\text{Br}(K_L \rightarrow \mu^+ \mu^-) = (6.84 \pm 0.11) \times 10^{-9}$ [9], the SM prediction is not reliable because of the long-distance effects. However, the dispersive method gives the estimate of the short-distance part as $\text{Br}(K_L \rightarrow \mu^+ \mu^-)|_{SD} < 2.5 \times 10^{-9}$ [38]. The short distance (SD) part can be calculated reliably and is on the same footing as $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ process except the difference of lepton line in the box diagram. The SD

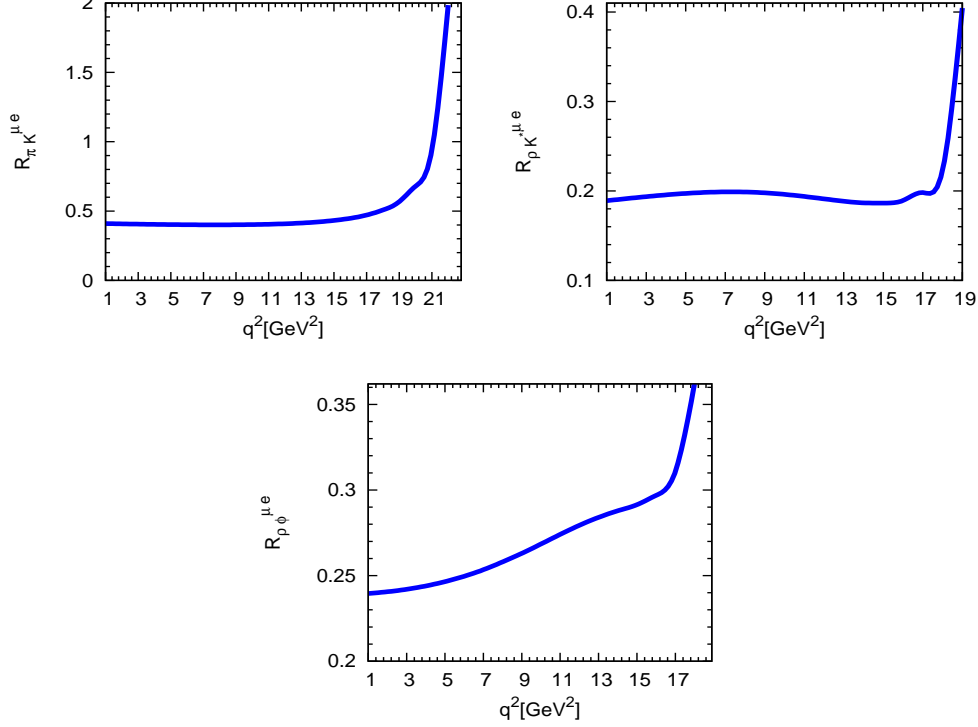


FIG. 8: The variation of observables $R_{\pi K}^{\mu e}$ (left panel), $R_{\rho K^*}^{\mu e}$ (right panel) and $R_{\rho \phi}^{\mu e}$ (bottom panel) with respect to q^2 in the scalar leptoquark model.

contribution to the effective Hamiltonian for $K_L \rightarrow \mu^+ \mu^-$ process in the SM is given by [39]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} (\lambda_c Y_{NL} + \lambda_t Y(x_t)) (\bar{s} \gamma^\mu (1 - \gamma_5) d) (\bar{\mu} \gamma_\mu (1 - \gamma_5) \mu), \quad (38)$$

where the functions Y_{NL} and $Y(x_t)$ are the contributions from charm and top quark respectively and the $Y(x_t)$ function in the next-to-leading order (NLO) is given as [41]

$$Y(x_t) = \eta_Y \frac{x_t}{8} \left(\frac{4 - x_t}{1 - x_t} + \frac{3x_t}{(1 - x_t)^2} \ln x_t \right). \quad (39)$$

The branching ratio for the SD part in the SM is

$$\text{BR}(K_L \rightarrow \mu^+ \mu^-)|_{\text{SD}} = \frac{G_F^2}{2\pi} \tau_{K_L} |V_{us}^* V_{ud}|^2 \left(\frac{m_\mu}{M_K} \right)^2 \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} f_K^2 M_K^3 |\Delta_{SM}^K|^2, \quad (40)$$

where

$$\Delta_{SM}^K = \frac{\alpha (\lambda_c Y_{NL} + \lambda_t Y(x_t))}{2\pi \sin^2 \theta_W V_{us}^* V_{ud}}. \quad (41)$$

Now using the interaction Lagrangian (4) for $X = (3, 2, 7/6)$ scalar leptoquark the relevant Hamiltonian of $K_{L,S} \rightarrow \mu^+ \mu^-$ process in the LQ model is given as

$$\mathcal{H}_{LQ} = \frac{\lambda_\mu^{21} \lambda_\mu^{22*}}{8M_Y^2} (\bar{s} \gamma^\mu (1 - \gamma_5) d) (\bar{\mu} \gamma^\mu (1 - \gamma_5) \mu). \quad (42)$$

Here we consider K_L as a pure CP-odd state and K_S as CP-even, which are decaying into CP-odd $\mu^+ \mu^-$ final state. If we include the contributions from both the K^0 and \bar{K}^0 decay amplitude, the leptoquark couplings given in Eq. (42) will be replaced by $\sqrt{2} \frac{\text{Re}(\lambda_\mu^{21} \lambda_\mu^{22*})}{M_Y^2}$ for $K_L \rightarrow \mu^+ \mu^-$ decay and for $K_S \rightarrow \mu^+ \mu^-$ process the coupling will be $\sqrt{2} \frac{\text{Im}(\lambda_\mu^{21} \lambda_\mu^{22*})}{M_Y^2}$. The decay processes $K_{L,S} \rightarrow \mu^+ \mu^-$ are studied in the leptoquark model in Refs. [24, 40]. Including the leptoquark contribution, the branching ratios of $K_L \rightarrow \mu^+ \mu^-$ process is given by

$$\begin{aligned} \text{BR}(K_L \rightarrow \mu^+ \mu^-) = & \frac{f_K^2 m_\mu^2 M_K}{8\pi} \tau_{K_L} \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \\ & \times \left| \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} (\lambda_c Y_{NL} + \lambda_t Y(x_t)) + \sqrt{2} \frac{\text{Re}(\lambda_\mu^{21} \lambda_\mu^{22*})}{8M_Y^2} \right|^2 \end{aligned} \quad (43)$$

and the value of the predicted branching ratio is presented in Table V. The masses of different particles and the life times of $K_{L,S}$ mesons have been taken from [9] and we use the same scaling law i.e. $\lambda^{ij} \simeq (m_i/m_j)^{1/4} \lambda^{ii}$ with $j > i$ to obtain the bounds on the required leptoquark couplings for $K_{L,S} \rightarrow \mu^+ \mu^- (e^+ e^-)$ transitions. Similarly we calculate the branching ratios of $K_S \rightarrow \mu^+ \mu^-$ and $K_{L,S} \rightarrow e^+ e^-$ processes and the corresponding values are listed in Table V. For the $K_L \rightarrow e^+ e^-$ mode, the experimental measurement $\text{Br}(K_L \rightarrow e^+ e^-) = (9_{-4}^{+6}) \times 10^{-12}$ [9] is in good agreement with the SM long-distance estimate $\text{Br}(K_L \rightarrow e^+ e^-)|_{LD} = (9 \pm 0.5) \times 10^{-12}$ [42], hence, the short distance contribution is almost negligible. The SM prediction for $K_S \rightarrow \mu^+ \mu^- (e^+ e^-)$ process is $2 \times 10^{-6} (8 \times 10^{-9}) \times \text{Br}(B_S \rightarrow \gamma\gamma) \sim 10^{-11} (10^{-14})$ respectively [43].

VII. $K_L \rightarrow \mu^\mp e^\pm$

Next, we would like to investigate the rare leptonic LFV $K_L \rightarrow \mu^\mp e^\pm$ decays. The effective Hamiltonian for $K_L \rightarrow \mu^+ e^-$ LFV decays in the $X = (3, 2, 7/6)$ scalar leptoquark model is

$$\mathcal{H}_{LQ} = \frac{\lambda_\mu^{22} \lambda_\mu^{11*}}{8M_Y^2} (\bar{s} \gamma^\mu (1 - \gamma_5) d) (\bar{\mu} \gamma^\mu (1 - \gamma_5) e), \quad (44)$$

TABLE V: The predicted branching ratios for $K_{L,S} \rightarrow \mu^+ \mu^- (e^+ e^-)$ processes (short-distance part) in the $X = (3, 2, 7/6)$ LQ model.

Decay processes	Predicted BR	Experimental values [9]
$K_L \rightarrow \mu^+ \mu^-$	$(0.13 - 2.2) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$
$K_L \rightarrow e^+ e^-$	1.1×10^{-12}	$9_{-4}^{+6} \times 10^{-12}$
$K_S \rightarrow \mu^+ \mu^-$	$< 2.23 \times 10^{-11}$	$< 9 \times 10^{-9}$
$K_S \rightarrow e^+ e^-$	$< 1.9 \times 10^{-15}$	$< 9 \times 10^{-9}$

and for $K_L \rightarrow \mu^- e^+$ process

$$\mathcal{H}_{LQ} = \frac{\lambda_\mu^{12} \lambda_\mu^{21*}}{8M_Y^2} (\bar{s} \gamma^\mu (1 - \gamma_5) d) (\bar{e} \gamma^\mu (1 - \gamma_5) \mu). \quad (45)$$

In the literature [40, 44] the LFV decay of kaon has been studied in the leptoquark and other new physics model. The corresponding branching ratio in the leptoquark model is given by

$$\begin{aligned} \text{BR}(K_L \rightarrow \mu^\mp e^\pm) &= \frac{f_K^2 \tau_{K_L}}{512\pi M_K^3} \left| \frac{\lambda_\mu^{22} \lambda_\mu^{11*} + \lambda_\mu^{12} \lambda_\mu^{21*}}{M_Y^2} \right|^2 \\ &\times \sqrt{(M_K^2 - m_\mu^2 - m_e^2)^2 - 4m_\mu^2 m_e^2} \left(M_K^2 (m_\mu^2 + m_e^2) - (m_\mu^2 - m_e^2)^2 \right). \end{aligned} \quad (46)$$

Now using the particle masses from [9] and the scaling ansatz for LQ couplings, the predicted branching ratios of $K_L \rightarrow \mu^\mp e^\pm$ process is

$$\text{BR}(K_L \rightarrow \mu^\mp e^\pm) = 7.17 \times 10^{-13}. \quad (47)$$

There exists only the upper limit on branching ratio of $K_L \rightarrow \mu^\mp e^\pm$ decay with value $\text{BR}(K_L \rightarrow \mu^\mp e^\pm) < 4.7 \times 10^{-12}$ at 90% C.L. [9] and our predicted result is within the experimental limit.

VIII. CONCLUSION

In this paper, we have studied the rare lepton flavour violating semileptonic B meson decays in the scalar leptoquark model. These decays are extremely rare in the SM as they occur at loop level. They are further suppressed due to the tiny neutrino masses in one of the loop. However, in the scalar leptoquark model, these decays can occur at the tree

level as the leptoquark couples to quark and lepton simultaneously thereby mediating the LFV processes at tree level. The scalar leptoquarks which do not have baryon number violation in the perturbation theory forbid proton decay and could be light enough to be accessible in accelerator searches. There are only two such leptoquarks $X(3, 2, 7/6)$ and $X(3, 2, 1/6)$ which could satisfy these conditions. We considered such leptoquarks and studied the various lepton flavour violating decays. The leptoquark parameter space is constrained using the recently measured branching ratios of $B_q \rightarrow l^+ l^-$ from LHCb and CMS experiments and using such constrained parameters we estimated the branching ratios of LFV decays such as $B^+ \rightarrow K^+(\pi^+)l_i^+ l_j^-$, $B^+ \rightarrow (K^{*+}, \rho^+)l_i^+ l_j^-$ and $B_s \rightarrow \phi l_i^+ l_j^-$. We study the ratios of various combination of LFV decays in order to check the presence of lepton non-universality. We also predicted the branching ratios of leptonic Kaon decays ($K_{L,S} \rightarrow \mu^+ \mu^-$) and the LFV $K_L \rightarrow \mu^+ e^-$ processes in the leptoquark model. We found that our predicted values are within the present experimental limits, the observation of which in the LHCb or upcoming Belle II experiments would provide unambiguous signal of new physics.

Acknowledgments

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